

Supermassive Black Holes from Primordial Black Hole Seeds

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The observational evidence for a population of quasars powered by supermassive black holes of mass $\gtrsim 10^9 M_\odot$ at redshifts $z \gtrsim 6$ poses a great challenge for any model describing the formation of galaxies. Assuming uninterrupted accretion at the Eddington limit, seed black holes of at least $1000 M_\odot$ are needed at $z \approx 15$. Here I study whether seeds could be primordial black holes (PBHs) which have been produced in the very early universe by the collapse of primordial density fluctuations. In particular, I study the expected number densities of PBHs in the relevant mass range for several classes of spectra of primordial density fluctuations and confront the results with observational data. While it seems to be possible to produce the required PBHs with spectra showing large enhancements of fluctuations on a certain scale, our hypothesis can be clearly disproved for a scale free spectrum of primordial fluctuations described by a power-law slope consistent with recent observations.

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I. INTRODUCTION

Supermassive black holes (SMBHs) are strongly believed to dwell in the centers of most galaxies. They are also thought to be the central engines of active galactic nuclei (AGN) and quasars [1]. While the above statements seem to be common knowledge, by now, the process of formation and subsequent evolution of these objects is one of the fundamental problems of contemporary astrophysics.

The problem concerning the seed black holes, which eventually grow to SMBHs, has sharpened during the last few years by some both accurate and interesting observations. First of all, in the local universe a strong correlation has been found between the mass of a SMBH and the velocity dispersion of the bulge of its host galaxy [2]. This provided evidence for the proposal that SMBHs have formed before and evolved together with the bulges of their host galaxies [3] and allowed to deduce comoving number densities for SMBHs of a given mass in the local universe.

On the other hand, a population of quasars with redshifts of $z \gtrsim 5.7$ (this value of z marks the onset of the so called *i*-dropout) has been found by the Sloan Digital Sky Survey (SDSS) [4, 5, 6]. Up to now twelve of these objects have been discovered and the most distant of them has a redshift $z = 6.4$ and a central black hole of mass $M \approx 3 \times 10^9 M_\odot$ [6]. This observation poses a great challenge for any model which seeks to describe the evolution of SMBHs from seed black holes by a combination of mergers of dark matter halos, which host SMBHs, and accretion of baryonic matter. The merging history of dark matter halos can be described in the hierarchical model of structure formation according to the so called extended Press-Schechter formalism [7]. However, after

a coalescence the newly formed SMBH gets a kick with some velocity which usually exceeds the escape velocity in a shallow dark matter halo. Thus the SMBH is cut off from its baryonic fuel supply sitting at the center of mass of the dark matter halo, and further growth of the black hole by accretion is no longer possible [8].

However, all models of growing SMBHs have to start off from a very early ($z \gtrsim 15$) population of seed black holes, each of them having a mass of at least $1000 M_\odot$. Astrophysical suggestions for the production of these black holes comprise the collapse of the entire baryonic cloud at the core of a dark matter halo, the remnants of an early generation of very heavy and metal poor stars (Population III), or the formation of an Intermediate Mass Black Hole (IMBH) from the collapse of an early star cluster. All these mechanisms which may be able to produce the seeds of the quasar population at $z \approx 2$ have serious problems to account for the black holes needed to have quasars as early as $z \approx 6$ (For a recent review of this complex, see, e.g., [9]).

As long as the origin of the mentioned black hole seeds is so unclear, one should have an open mind to alternative scenarios. Here I study whether these seeds can be primordial black holes (PBHs) produced during the very first of cosmic evolution from the gravitational collapse of a sufficiently large overdense region [10]. For a given spectrum of primordial fluctuations, which may arise, e.g., from some inflationary scenario, the expected mass spectrum of PBHs can be computed along the lines of, e.g., ref. [11]. The fluctuation spectrum is known to some degree for scales relevant for CMB and LSS observations [12, 13], but has to be extrapolated to smaller scales in order to compute the abundances of PBHs. It is the goal of this work to study whether the seeds of the $z \approx 6$ quasars can be PBHs which arose upon the collapse of suitably parametrized primordial fluctuations consistent with recent observations.

The paper is organized as follows: in the next section I describe shortly the problem of growing a SMBH by ac-

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cretion from a seed black hole. Then I shall discuss the formation of PBHs through the collapse of an overdense region in the very early universe, together with some possible parametrizations of the spectrum of primordial curvature perturbations. In the following section, I study under which circumstances the resulting PBHs could be the seeds for the early population of quasars mentioned above. In the last section, I will discuss the results and draw my conclusions.

II. ACCRETIONAL GROWTH OF SMBHS

In this section, I give a simple estimation of the time needed to build a SMBH from a black hole seed by accretion. I will not go into details of the accretion process, but one should have in mind that the scenario presented here is highly optimistic, and so the calculated evolution times should be seen as *lower* limits to the real ones.

Let us assume that a seed black hole of initial mass M_i sits at the center of some dark matter halo and starts at time t_i to accrete radiatively baryonic matter with a rate \dot{m}_{fuel} at the Eddington limit with an efficiency ε . The luminosity of the radiation is then given by[33]

$$L = \varepsilon \dot{m}_{\text{fuel}} c^2 = 1.15 \frac{4\pi G c m_p}{\sigma_T} M_{\bullet}, \quad (1)$$

where m_p denotes the proton mass, σ_T the Thomson cross section for an electron, M_{\bullet} the (evolving) mass of the black hole, and the factor 1.15 is the mean atomic weight per electron for the hydrogen and helium gas mixture. Under the assumption that all accreted mass energy, which is not radiated away, feeds the black hole[34], the SMBH grows according to the equation

$$\dot{M}_{\bullet} = (1 - \varepsilon) \dot{m}_{\text{fuel}}, \quad (2)$$

which, upon inserting eq. (1), can be easily integrated to yield the solution

$$M_{\bullet}(t) = M_i \exp \left[\frac{t - t_i}{\tau} \right], \quad (3)$$

where the *e*-folding time τ is given by

$$\tau = 1.24 \times \frac{\varepsilon}{1 - \varepsilon} \times 10^{16} \text{ s} \approx 3.9 \times \frac{\varepsilon}{0.1} \times 10^7 \text{ yrs}. \quad (4)$$

The *e*-folding time is, of course, an increasing function of ε , which itself is an increasing function of the angular momentum of the black hole and can reach values up to 0.4 for a maximal rotating hole. So the value $\varepsilon \approx 0.1$ of this so called thin-disc accretion mode, which is adopted here and also in a number of other papers on this subject, seems to be quite optimistic for our goal to obtain short growing times for SMBHs. The assumption that thin disc accretion plays a major role in SMBH growth obtained further confidence recently by evaluating a relation which is known as Soltan's argument [14]: integrating the (measured) quasar luminosity function over all

wave lengths and redshifts under the assumption of thin-disc accretion yields an estimate of today's mass density in SMBHs. This density turned out to be consistent with the density of the local SMBH population [15].

Other accretion modes like the low effectiveness advection dominated accretion or the low efficiency super Eddington mode seem to suffer from considerably high outflows, possibly connected to jets, or instabilities which prevents them from being effective for a sufficiently long time. However, super Eddington accretion may be important for some part of the early accretion history [16]. But even if this should be the case, because of large outflows the process of SMBH growth is not expected to proceed faster than in the thin-disc mode.

Our goal is to build up a SMBH of mass $M_{\bullet} = 3 \times 10^9 M_{\odot}$ at $z = 6.4$ from a black hole seed of mass M_i . As initial time for the accretion process we take a redshift $z = 15$, which is motivated by the onset of reionization [17]. For a flat ΛCDM -cosmology with $H_0 = 70 \text{ km}/(\text{s} \cdot \text{Mpc})$ and $\Omega_M = 0.3$, we obtain a growing time of $t(z = 6.4) - t(z = 15) = 0.84 \text{ Gyr} - 0.27 \text{ Gyr} = 0.57 \text{ Gyr}$. Then from eq. (3), we get a minimal seed black hole mass $M_i \approx 1300 M_{\odot}$. Here we have assumed that the accretion process is effective without any interruptions. It is often assumed that accretion occurs during certain cycles triggered by a larger merger (see, e.g., [18]). These periods are then characterized by some duty cycle (usually $10^7 - 10^8$ years) or the amount of baryonic matter available for SMBH growth. Again our model seems to be quite optimistic, but one should also have in mind that at very early times the supply of baryonic matter is larger than after the onset of significant star formation.

As mentioned above it is very challenging to provide an astrophysical mechanism to produce black holes of this mass at such an early time in the cosmic evolution [8]. The collapse of the entire baryonic cloud of a dark matter halo seems to be problematic due to fragmentation after H₂-cooling. So the first collapsed baryonic objects seem to be a population of superheavy (up to $1000 M_{\odot}$) stars, which, depending on their mass, collapse further to a black hole or explode via a pair instability supernova without leaving any remnants [19]. The latter outcome would pollute the environment with metals, which trigger the more effective atomic line cooling mechanism and so prevent the formation of further superheavy stars. Even if the circumstances would favor the formation of black holes it seems to be very problematic to have them with the desired mass at such an early time (270 million years) of the cosmic evolution.

This problem gives us the motivation for investigating whether the seed black holes for the formation of SMBHs could be primordial.

III. PRIMORDIAL BLACK HOLES

In this section we provide the formulae to calculate the mass spectrum for primordial black holes (PBHs),

which are formed by the collapse of primordial density fluctuations, and discuss some ways to parametrize the spectrum of these fluctuations.

A. PBH formation

Primordial black holes can be formed by the collapse of primordial density fluctuations [10]. Because the Jeans radius in the radiation dominated era is of the order of the horizon scale, the mass of such a PBH is about the horizon mass M_H at the time of formation. Now the collapse of an overdense region is only possible, if the rms of the primordial fluctuations there, averaged over a Hubble volume, is larger than a threshold δ_{\min} .

The value of δ_{\min} is and has been a matter of some discussion. For a long time it had been thought to be about 1/3. At the end of the nineties numerical simulations suggested that $\delta_{\min} \approx 0.7$ would be more appropriate [20]. Very recently, an analytic calculation which employs peaks theory instead of modified Press-Schechter theory has been proposed which arrives at the result that both analytic approaches are in good agreement if one takes the Press-Schechter value $\delta_{\min} \approx 0.3 - 0.5$ [21]. In the present work we take $\delta_{\min} = 0.6$ while keeping in mind that our results and constraints could be tightened or loosened by a different choice.

A fluctuation mode with some comoving wave number k enters the horizon at time t_k defined by $k = a(t_k)H(t_k)$. The probability for the formation of a PBH of mass $M_H(t_k)$ at time t_k is then given by

$$\beta(M_H) \approx \frac{\sigma_H(t_k)}{\sqrt{2\pi}\delta_{\min}} e^{-\frac{d_{\min}^2}{2\sigma_H^2(t_k)}}, \quad (5)$$

where $\sigma_H^2(t_k)$ denotes the variance of primordial density fluctuations at time t_k , averaged over one Hubble volume at that time. It is given by [11]

$$\sigma_H^2(t_k) = \frac{8}{81\pi^2} \int_0^{k_e/k} dx x^3 \Delta_R^2(kx) T^2(kx, t_k) W_{TH}^2(x). \quad (6)$$

Here, $\Delta_R^2(k)$ is the spectrum of primordial curvature fluctuations, $W_{TH}(x)$ is the Fourier transform of the top head window function given by

$$W_{TH}(x) = \frac{3}{x^3} (\sin x - x \cos x), \quad (7)$$

and $T^2(k, t)$ is the transfer function for the subhorizon evolution of the density fluctuations. In the above integral we are interested in the function $T^2(kx, t_k)$, which turns out to be $W_{TH}^2(c_s x)$, where $c_s = 1/\sqrt{3}$ denotes the speed of sound in the radiation dominated era. The scale k_e is an ultraviolet cut off for small scales which is needed if one chooses a fluctuation spectrum for which the integral (6) does not converge. Here one usually takes the scale of reheating. However, in the calculations of the present work the integral always converges, and k_e can

be safely taken to be infinity or, more convenient for numerical calculations, some value where the integrand is sufficiently small.

Now we deduce the relations between various quantities for PBHs of mass $M_H(t_k)$. The horizon mass is defined to be

$$M_H(t_k) = \frac{4\pi}{3} \rho(t_k) \left(\frac{1}{H(t_k)} \right)^3 = \frac{t_k}{G} \equiv 2.0 \times 10^5 \frac{t_k}{1s} M_\odot. \quad (8)$$

We are interested in PBHs of about $1000 M_\odot$. From eq. (8) we see that these are produced well before the era of nucleosynthesis, but when we are dealing with fluctuation spectra we must ensure that no PBHs much heavier than the ones mentioned are produced, because the fluctuations from which they result could spoil the outcome of nucleosynthesis. We will take care of this, but it will not be mentioned in the following. However, on the other hand, recent CMB observations allowed to measure the baryon density of the universe directly, and some subtle strains between this value and the predictions of standard nucleosynthesis have shown up [22]. Maybe one reason, among others, could be a certain amount of fluctuations of the order of the horizon scale at this time.

For the temperature T_k at the time of formation t_k , we obtain

$$\begin{aligned} H^2(t_k) &= \frac{1}{4t_k^2} = \frac{8\pi G}{3} \frac{\pi^2}{30} g(t_k) T_k^4 \\ \Rightarrow k_B T_k &= \frac{1.6}{\sqrt[4]{g(t_k)}} \sqrt{\frac{1s}{t_k}} \text{ MeV}. \end{aligned} \quad (9)$$

Here $g(t_k)$ denotes the (effective) number of relativistic degrees of freedom at time t_k . During the formation of the PBHs we are interested in, these degrees comprised electrons, positrons, photons, and three kinds of left-handed neutrinos, so we always take $g(t_k) = 10.75$ in our calculations.

The connection between the horizon mass and the scale of the horizon is given by the formula

$$\begin{aligned} M_H(t_k) &= \frac{4\pi}{3} \rho(t_k) \frac{1}{H(t_k)} \left(\frac{a(t_k)}{a(t_k)H(t_k)} \right)^2 \\ &= \frac{4\pi}{3} \sqrt{\frac{3\rho(t_k)}{8\pi G}} \left(\frac{a(t_k)}{k} \right)^2 = \frac{4\pi}{3} \sqrt{\frac{\pi g(t_k)}{80G}} \left(\frac{a(t_k)T_k}{k} \right)^2 \\ &= \frac{\pi}{3} \sqrt{\frac{\pi g(t_k)}{5G}} \left(\frac{4}{11} \right)^{2/3} \left(\frac{T_0}{k} \right)^2 \\ \Rightarrow M_H(t_k) &= 6.3 \times 10^{12} \sqrt{g(t_k)} \left(\frac{1 \text{Mpc}^{-1}}{k} \right)^3 M_\odot. \end{aligned} \quad (10)$$

At last, for the comoving number density of PBHs of mass $M_H(t_k)$ we get

$$n(M_H(t_k)) = \frac{\beta(t_k)\rho(t_k)}{M_H(t_k)} \left(\frac{a(t_k)H(t_k)}{H(t_k)} \right)^3 \quad (11)$$

$$\begin{aligned}
&= \frac{3\beta(t_k)k^3}{8\pi GH(t_k)M_H(t_k)} = \frac{3\beta(t_k)k^3 2t_k}{8\pi GM_H(t_k)} = \frac{3\beta(t_k)k^3}{4\pi} \\
&\equiv 3.8 \times 10^{18} g(t_k)^{3/4} \beta(t_k) \left(\frac{M_\odot}{M_H(t_k)} \right)^{3/2} \text{Mpc}^{-3}.
\end{aligned}$$

It is this formula, together with (5) and (6), which we will employ in our considerations. As an input we need suitable parametrizations of the primordial fluctuation spectrum. A few of them are discussed in the following subsection.

B. Parametrizations of $\Delta_R^2(k)$

There are several possibilities to parametrize the spectrum of primordial curvature perturbations $\Delta_R^2(k)$. Fits of observations of LSS and CMB usually employ a power-law spectrum with some amount of running:

$$\begin{aligned}
\Delta_R^2(k) &= \Delta_R^2(k_0) \left(\frac{k}{k_0} \right)^{n-1} \\
n &= n_0 + \frac{1}{2!} n_1 \ln \left(\frac{k}{k_0} \right) + \frac{1}{3!} n_2 \ln^2 \left(\frac{k}{k_0} \right) + \dots
\end{aligned} \tag{12}$$

Here k_0 is some pivot scale, and the parameters n_0 and n_1 denote the tilt and the running of tilt at that scale, respectively:

$$\begin{aligned}
n_0 &= \frac{d \ln \Delta_R^2}{d \ln(k/k_0)}(k_0) \equiv n_S(k_0), \\
n_1 &= \frac{dn_S}{d \ln(k/k_0)}(k_0) \equiv \alpha_S(k_0).
\end{aligned} \tag{13}$$

The simplest models of inflation suggest that the coefficients n_i scale as powers ε^i of some slow-roll parameter ε [23]. In this case, for $\varepsilon \approx 0.1$ an expansion up to $i = 4$ can be expected to be accurate to 10% for about 16 e -foldings around the pivot scale. For a pivot scale of 0.05 h/Mpc , this means sensitivity down to horizon masses of $400M_\odot$. For general n_i this parametrization can, of course, also be seen as a phenomenological, model independent description of the primordial perturbations.

From the combined analysis of several CMB and Large Scale Structure (LSS) observations the WMAP team claimed to have detected a value of $\alpha_s = -0.075^{+0.044}_{-0.045}$ [12]. The large modulus of this value is quite constraining for simple models, and the significance of the observation has been doubted by several authors. However, recent small scale measurements of the CMB by the Cosmic Background Imager (CBI) yielded further evidence

for a negative running parameter $\alpha_S = -0.087^{+0.028}_{-0.028}$ [24]. A definitive measurement of α_S , and possibly also of n_2 and/or n_3 , is expected from the Planck satellite in combination with the full data set of the SDSS and other upcoming LSS surveys.

The spectrum may also be expanded directly in powers of the logarithm of the comoving scale

$$\Delta_R^2(k) = \Delta_R^2(k_0) \left(1 + \sum_{i \geq 1} \frac{a_i}{i!} \ln^i \left(\frac{k}{k_0} \right) \right). \tag{14}$$

This expansion is not suggested by some specific model, but it seems to be more appropriate for predictions of some more general inflationary models and gives in many cases a better reconstruction of the ‘real’ spectrum than (12), cf. [25]. It is clear that cosmological parameter fits should be done with a few different parametrizations as long as one has no knowledge of the form of the spectrum which is realized in nature. Even very accurate data may lead to a bad fit if the employed parametrization is not suitable. Note that a similar problem occurs when one tries to model the time evolution of the equation of state of dark energy.

The a_i can – for fixed k_0 – be expressed in terms of the n_i in the following way:

$$\begin{aligned}
a_1 &= n_0 - 1, \quad a_2 = (n_0 - 1)^2 + n_1, \\
a_3 &= (n_0 - 1)^3 + 3(n_0 - 1)n_1 + n_2, \quad a_4 = (n_0 - 1)^4 \\
&\quad + 6(n_0 - 1)^2 n_1 + 4(n_0 - 1)n_2 + 3n_1^2 + n_3, \dots
\end{aligned} \tag{15}$$

However, one should in general not expect that independent fits of the spectra (12) and (14) up to a given order yield the relations (15). This may happen because the number of expansion terms needed to reproduce the real spectrum over a certain range of scales is probably different for parametrizations like (12) and (14).

We now come to a more theory driven and model dependent parametrization of $\Delta_R^2(k)$. Here it is assumed that the fluctuations are generated during an inflationary era described by an inflaton potential $V(\phi)$, the derivative of which has a jump at some value ϕ_s corresponding to a comoving wave number k_s . It has been shown by Starobinsky [26] that the ensuing fluctuation spectrum has a universal form around the scale k_s , which can be calculated exactly and is given by

$$\Delta_R^2(k) = \frac{\Delta_0^2}{p^2} \left\{ 1 - \frac{3}{x} (p-1) \left[\left(1 - \frac{1}{x^2} \right) \sin 2x + \frac{2}{x} \cos 2x \right] \right\} \tag{16}$$

$$+ \frac{9(p-1)^2}{2x^2} \left[1 + \frac{1}{x^2} \right] \left[1 + \frac{1}{x^2} + \left(1 - \frac{1}{x^2} \right) \cos 2x - \frac{2}{x} \sin 2x \right] \} .$$

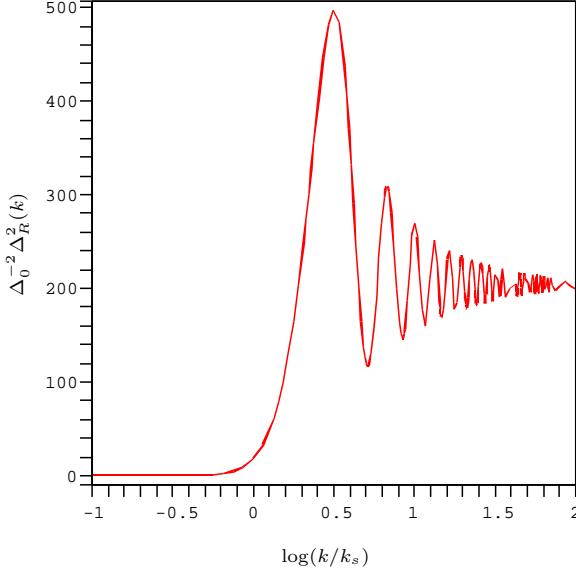


FIG. 1: (Color online) Typical form of the BSI spectrum with jump parameter $p = 0.07$. The jump scale k_s is a free parameter and the large scale normalization may be chosen consistent with observational results.

Here $x = k/k_s$ denotes the wave number in units of k_s and p is the ratio of the left and right handed limits of the derivative of the inflaton potential at ϕ_s . The typical form of the spectrum is depicted in Fig. 1. Because of the broken scale invariance we will refer to (16) as the BSI-spectrum. Its asymptotic behavior is given by

$$\Delta_0^2 \xrightarrow{x \rightarrow 0} \Delta_R^2(k) \xrightarrow{x \rightarrow \infty} \frac{\Delta_0^2}{p^2} \quad (17)$$

and approaches the scale invariant Harrison-Zel'dovich spectrum on large and small scales, but with different amplitudes. If the jump in the spectrum would occur at larger scales it could perhaps be observable as some enhancement of the small scale structure of galaxy distributions, which would be in conflict with actual observations. However, the scales we are interested in here are too small to be problematic with respect to small scale structure.

The normalization Δ_0^2 may be taken from CMB measurements, and thus the model has two free parameters, p and k_s . But even if such a kind of spectrum is realized in nature, one would expect additional structure superimposed on it on large scales which depends on the actual form of the inflaton potential.

IV. COMPUTATION OF PBH ABUNDANCES

Now we want to apply the formulae of the last section to see whether PBHs can contribute to the population of seed black holes which then evolve to become SMBHs. In [6] the comoving number density of the population of $z \gtrsim 6$ quasars has been estimated to be $(6.4 \pm 2.4) \times 10^{-10} \text{ Mpc}^{-3}$ (based on a population of 10 objects). It is not easy to get accurate constraints for the slope of the luminosity function at such an early era, but one should expect these objects to be the heaviest ones at their time with masses up to about $3 \times 10^9 M_\odot$. In section two, we have shown that black hole seeds with a mass of more than $1300 M_\odot$ at $z \approx 15$ are required to grow the SMBHs powering these quasars. In order to test the hypothesis whether these seeds could be primordial, we try to reproduce this density with PBHs. Taking into account some inefficiencies in the accretion and merging processes involved, we shall study whether number densities few orders of magnitude higher than the density mentioned above seem to be possible. For the masses it should be sufficient to have the peak of the distribution at a value of several $100 M_\odot$. A PBH of this mass can form a small overdense region of its own which is able to grow and take part in merging events after the onset of the matter dominated era, according to the hierarchical model of structure formation. After the nonlinear collapse of the halo the accretion of baryonic matter could start very early and does not have to wait until seed black holes have been formed by astrophysical processes having their own problems. Another mode of early growth could be the “accretion” of energy from a surrounding quintessence field[35] [27] or the swallowing of interacting dark matter. Thus, it seems to be possible to have seed black holes of the desired mass at $z \approx 15$ from PBHs which are slightly lighter, and we will be content if the latter have masses of several $100 M_\odot$, as mentioned above.

We start with a very simple spectrum of primordial curvature fluctuations which, nevertheless, is up to now consistent with all observations: a scale free power law

$$\Delta_R^2(k) = 2.95 \times 10^{-9} A(k_0) \left(\frac{k}{k_0} \right)^{n_S-1} . \quad (18)$$

A fit of several recent CMB and LSS observations at the pivot scale $k_0 = 0.05 \text{ Mpc}^{-1}$ yields[36] $A = 0.631^{+0.020}_{-0.019}$ and $n_S = 0.966^{+0.025}_{-0.020}$ [13]. Putting these values in the formulae of the preceding section gives – as could be expected – a rather disappointing result: The expected number density of black holes is absolutely negligible. This is not surprising given the known similar results of

attempts to establish a significant contribution of dark matter from PBHs [11]. So we arrive at our first result: *If the spectrum of primordial curvature fluctuations from scales relevant for CMB and LSS up to $k \approx 10^5 \text{ Mpc}^{-1}$ is given by a scale free power law with parameters consistent with CMB and LSS fits, then the seed black holes for future SMBHs cannot be primordial.*

It does not make much sense to ask which constant parameter n_s would be needed in order to produce an appreciable amount of PBHs with the desired mass for the following reason: The number density is in this case a sharply *decreasing* function of the PBH mass, and so one runs very quickly into an overproduction of small mass PBHs. These would overclose the universe for $n_s \gtrsim 1.35$ [28]. If one cuts off all fluctuations on scales smaller than the ones needed to produce PBHs of about $1000 M_\odot$, a value of $n_s \approx 2.1$ would be required to produce the desired number of black hole seeds.

It is clear that an extrapolation of a scale free power law over such a large range is highly questionable. Note that this remark applies in particular to the attempts to compute dark matter abundances with such a spectrum, because the PBH masses which seem to be preferred in such studies are of the order $10^{15} - 10^{20} \text{ g}$ and are therefore related to scales much smaller than the ones considered in the present work.

Let us now take some more sophisticated parametrizations of primordial curvature fluctuations. We start with the “running tilt”-spectrum (12) with a pivot scale of $k_0 = 0.05 \text{ Mpc}^{-1}$ and values of the normalization constant as well as n_0 and n_1 consistent with cosmological fits. After some trial and error we see that it is possible to find values for the remaining two parameters which describe a fluctuation spectrum that is able to produce a reasonable number of PBHs of the desired mass. The result can be seen in Fig. 2, where we have plotted the expected comoving number density of PBHs per e -folding in M , which is more intuitive than the spectral number density. Although the adopted parameter values do not seem to be completely unphysical, one should be careful to give them (or the amplitude and scale of the bump in the fluctuation spectrum they lead to) too much relevance. We only want to show that such a spectrum is still possible. It is important to note that it should be feasible for future observations to obtain a quite accurate value for n_1 and possibly more or less strong constraints on n_2 . The contributions of the terms of order five and larger in the expansion of the fluctuation spectrum are irrelevant for the scales we are interested in, provided, of course, that the parameters are of the order of unity at most. Since the resulting PBH density is quite sensitive to n_2 , it might be possible to falsify the idea of having PBHs as SMBH seeds.

One might think that the message of the previous paragraph is by no means surprising, and that for any kind of parametrization of primordial fluctuations some clever

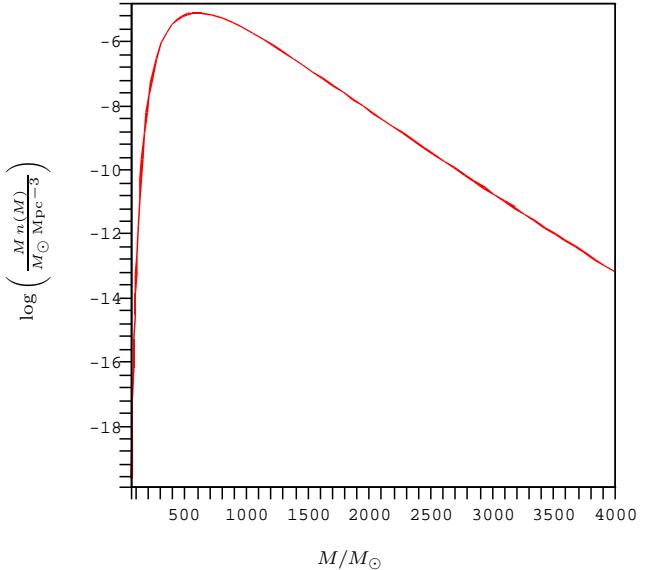


FIG. 2: (Color online) Expected comoving number density of PBHs per e -folding in M for the spectrum of primordial curvature fluctuations (12) with adopted parameters $n_0 = 0.98$, $n_1 = -0.05$, $n_2 = 0.123$, $n_3 = -0.022$, and all other n_i equal to zero.

choice of parameters could be found which does the desired job. However, this is *not* the case as can be seen in the example of the parametrization (14). Because we are not aware of any published fit of its parameters, we will take the values $a_1 = -0.03$ and $a_2 = -0.05$ in the following, which are suggested by the relations (15). Playing the same game as above, it turns out to be impossible to find satisfying higher order parameters to produce an appreciable amount of PBHs. More precisely, we need more parameters than can ever be sensibly fitted, and the values of these parameters are unacceptably large. This means that if the parametrization (14), together with the values of a_1 and a_2 mentioned above, should ever lead to a good fit of the primordial fluctuations, then PBHs are strongly disfavored to be the seeds of SMBHs.

The reason for this is that we have assumed the first coefficients a_1 and a_2 to be negative. Thus less fluctuations are expected on small scales than on large scales. To produce PBHs we have to achieve a turnover for which quite large positive values of the following parameters are needed. But, unfortunately, these overproduce PBHs on smaller scales and so further negative parameters have to be introduced. Although the parametrization (12) is also plagued by this problem, it turns out to be not so dramatic as in the present case because there the expansion is performed in the exponent.

The conclusions of the preceding paragraph relied crucially on the fact that the parameters a_1 and a_2 were

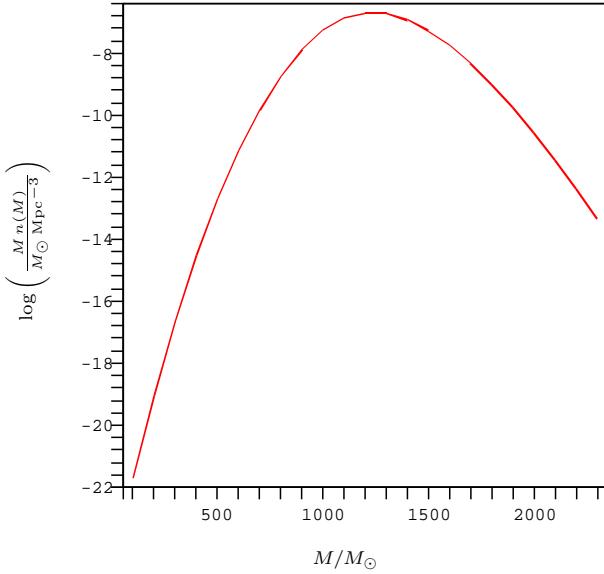


FIG. 3: (Color online) Expected comoving number density of PBHs per e -folding in M for the BSI-spectrum of primordial curvature fluctuations (16) with adopted parameters $p = 0.0004$, $M_s = 2000M_\odot$, and large scale normalization $\Delta_0^2 = 3 \times 10^{-9}$.

assumed to be negative. However, present observations are not yet able to exclude positive values for them completely, and in this case it would be easier to extend a good fit of (14) to a spectrum suitable to produce a considerable number of PBHs, but, of course, a good fit of (12) with a positive value of α_S would still be more favorable for our intentions.

Now we come to the parametrization of the primordial fluctuations according to the BSI-spectrum (16). Here we have only two parameters, p and k_s , at our disposal, and it seems to be much more difficult to produce PBHs out of this spectrum. But in practice things turn out not to be that worse because of the following reason: what we need is a spectrum which is compatible with observations on large scales, shows a large bump at a scale suited for producing PBHs of the desired mass, and has less fluctuations on smaller scales in order to avoid an overproduction of small PBHs. As can be seen in Fig. 1, the BSI-spectrum has such a form, and the parameters p and k_s are exactly the screws for the position and the amplitude of the mentioned bump. It should be noted further that, contrary to the preceding parametrizations, the spectrum is fixed to its known large scale behavior by only one parameter, namely the overall normalization.

A result of our juggling with the parameters can be seen in Fig. 3. Instead of the scale k_s we have introduced via eq. (10) the PBH mass M_s connected with this scale

as a more convenient parameter. The maximum of the distribution shows up at a slightly lower scale than M_s , which is not surprising because the bump in the fluctuation spectrum also appears at scales smaller than k_s (see Fig. 1).

Although it seems to be possible to obtain the desired number density of PBHs with the BSI-spectrum, it should be noted that in Fig. 3 we have chosen for the large scale normalization of the fluctuation spectrum a value which is situated at the upper end of its confidence region and, more important, the adopted value $p = 0.0004$ leads to fluctuations of the order 1% at the bump, which itself is quite narrow. It does not make sense to consider smaller values of p in our calculations, because then linear theory would break down and we could not trust our results. Furtheron, it is hard to accept such fluctuations from a realistic point of view.

The BSI-spectrum could be further constrained by analyzing specific inflaton potentials, which would lead to further structure in addition to the universal part of the spectrum, in particular on large scales, where it could be better anchored to observations.

V. CONCLUSIONS

We have studied whether the black hole seeds needed to grow SMBHs could be primordial ones formed within the very first second of cosmic evolution. In order to do this the expected number densities for PBHs of the relevant mass ($\approx 1000 M_\odot$) have been calculated for a few parametrizations of the spectrum of primordial curvature fluctuations normalized to their known large scale values. As expected, it is impossible to produce the required black holes with a scale free spectrum of the form (18) without a huge overproduction of smaller PBHs which eventually closes the universe.

What is required, is a fluctuation spectrum which is consistent with observations on large scales, shows a large bump with amplitudes up to several parts of a percent on scales of about 0.1 pc^{-1} , and decreases quickly on smaller scales in order to avoid an overproduction of PBHs. This became clear in the considerations of three different parametrizations which seem to be capable of achieving the mentioned demands. Two of them were expansions in powers of $\ln k$. In the first one the expansion was done for the exponent of a power law, and in the second one for the fluctuation spectrum itself. Two parameters, the overall normalization and the power law slope at some large pivot scale, can be fixed quite accurately with present observations. A third one is only vaguely constrained today. It is, of course, possible to reproduce the desired form of the spectrum with an arbitrary number of parameters, but if one restricts the considerations to parametrizations which employ only a small number of parameters in addition to the ones fixed by observations and postulates further that their modulus should not be larger than about unity, as is suggested

by inflationary models, than the scenario of having PBHs as SMBH seeds could be strongly disfavored in the near future. This will be the case for the second parametrization if the evidence of a negative value of the running parameter α_S should be confirmed.

The third parametrization, the BSI-spectrum, on the other hand, employed only two adjustable parameters and one parameter, the overall normalization, to anchor it to its large scale value. As a universal spectrum it cannot be further constrained by observations, but it is clear that there must be some superimposed structure on top of it, which arises from the specific form of the inflaton potential. This might be delineated by future observations, and so the position of the characteristic scale of the spectrum could be constrained, which then perhaps leads us to abandon that model.

It seems to be quite difficult to construct viable physical models which produce the desired jump in the spectrum of primordial fluctuations. However, we have already seen that the BSI model does the job. Other scenarios are, e.g., the formation of domain walls during inflation [29], one or more phase transitions during inflation [30], or some events of resonant particle production in the inflationary era [31]. Most of these mechanisms have been invented as a mean to produce features in the primordial fluctuation spectra which are detectable by, e.g., CMB, LSS or Ly- α observations. However, there seems to be no reason why they should not be applicable on smaller scales, which are relevant for the considerations of the present work.

It should be noted that the required scale of the bump in the fluctuation spectra is near the scale relevant for fluctuations triggered by the quark-hadron phase transition [32]. However, the PBHs resulting from this fluctuations are expected to be lighter than $\approx 1 M_\odot$, and so a larger amount of early growth would be required, perhaps by the absorption of a quintessence field [27].

Future observations and experiments promise to be able to decide whether PBHs are serious candidates for SMBH seeds. High energy physics may shed some light on the specific mechanism of inflation, in particular the on scale of reheating. CMB, LSS, and weak lensing observations are expected to proceed in reconstructing the spectrum of primordial curvature fluctuations to smaller scales. The epoch of galaxy formation and reionization will be reached with the next generation of telescopes, the space based gravity wave detector LISA will clarify the abundance and importance of mergers of SMBHs, and further supernovae observations should explore the evolution of dark energy, which could lead to a reassessment of the formation times of the early generation of quasars.

Thus, primordial black holes, which have been a matter of debate for decades, may, at the end, show up at places where they had hardly been expected!

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[1] D. Lynden-Bell, *Nature*, **223**, 690 (1969).
[2] L. Ferrarese and D. Merritt, *Astron. J.* **539**, L9 (2000); K. Gebhardt et al. , *Astrophys. J.* **539**, L13 (2000); S. Tremaine et al. , *Astrophys. J.* **574**, 740 (2002).
[3] J. Silk and M.J. Rees, *Astron. Astrophys.* **331**, L1 (1998).
[4] D.G. York et al. , *Astron. J.* **120**, 1579 (2000).
[5] K. Abazajian et al., *Astron. J.* **126**, 2081 (2003).
[6] X. Fan et al. , *Astron. J.* **122**, 2833 (2001); *Astron. J.* **125**, 1649; astro-ph/0405138 ; C.J. Willott, R.J. McLure and M.J. Jarvis, *Astrophys. J.* **587**, L 15 (2003).
[7] W.H. Press and P.L. Schechter, *Astrophys. J.* **187**, 425 (1974); C. Lacey and S. Cole, *MNRAS* **262**, 627 (1993).
[8] Z. Haiman and E. Quataert in “Supermassive Black Holes in the Distant Universe”, ed. A.J. Barger, Kluwer Academic Publishers (2004) or astro-ph/0403225.
[9] Z. Haiman, astro-ph/0404196.
[10] Ya.N. Zel'dovich and I.D. Novikov, *Sov. Astron.* **10**, 602 (1967); S.W. Hawking, *Mon. Not. R. Ast. Soc.* **152**, 75 (1971); B.J. Carr and S.W. Hawking, *Mon. Not. R. Ast. Soc.* **168**, 399 (1974); B.J. Carr, *Astrophys. J.* **205**, 1 (1975); I.D. Novikov, A.G. Polnarev, A.A. Starobinsky, and Ya.N. Zel'dovich, *Astron. Astrophys.* **80**, 104 (1979).
[11] D. Blais, T. Bringmann, C. Kiefer, and D. Polarski, *Phys. Rev.* **D67**, 024024 (2003).
[12] H.V. Peiris et al. , *Astrophys. J. Suppl.* **148**, 213 (2003).
[13] M. Tegmark et al. , *Phys. Rev.* **D69**, 103501 (2004).
[14] A. Soltan, *Mon. Not. R. Ast. Soc.* **200**, 115 (1982).
[15] Q. Yu and S. Tremaine, *Mon. Not. R. Ast. Soc.* **335**, 965 (2002).
[16] T. Kawaguchi et al. , astro-ph/0405024.
[17] D.N. Spergel et al. , *Astrophys. J. Suppl.* **148**, 175 (2003).
[18] J.M. Bromley, R.S. Somerville, and A.C. Fabian, *Mon. Not. R. Ast. Soc.* **350**, 456 (2004).
[19] T. Abel, G.L. Bryan and M.L. Norman, *Astrophys. J.* **540**, 39 (2000); *Science* **295**, 93 (2002); V. Bromm, P.S. Coppi and R.B. Larson, *Astrophys. J.* **527**, 5 (1999); *Astrophys. J.* **564**, 23 (2002); A. Heger et al. , *Astrophys. J.* **591**, 288 (2003).
[20] J.C. Niemeyer, and K. Jedamzik, *Phys. Rev. Lett.* **80**, 5481 (1998); *Phys. Rev. D* **59**, 124013 (1999).
[21] A.M. Green et al. , astro-ph/0403181.
[22] R.H. Cyburt, B.D. Fields, and K.A. Olive, *Phys. Lett.* **B567**, 227 (2003).
[23] A.R. Liddle and D.H. Lyth, “Cosmological Inflation and Large-Scale Structure”, Cambridge University Press (2000).
[24] A.C.S. Readhead et al. , astro-ph/0402359.
[25] S.M. Leach, A.R. Liddle, J. Martin and D.J. Schwarz, *Phys. Rev.* **D66**, 023515 (2002).
[26] A.A. Starobinsky, *JETP Lett.* **55**, 489 (1992).
[27] R. Bean and J. Magueijo, *Phys. Rev.* **D66**, 063505 (2002).
[28] T. Bringmann, C. Kiefer, and D. Polarski, *Phys. Rev.* **D65**, 024008 (2002).

- [29] M.Yu. Khlopov, S.G. Rubin, and A.S. Sakharov, astro-ph/0401532.
- [30] J.A. Adams, G.G. Ross, and S. Sarkar, Nucl. Phys. **B 503**, 405 (1997); S.G. Rubin, A.S. Sakharov, and M.Yu. Khlopov, J. Exp. Theor. Phys. **91**, 921 (2001).
- [31] Ø. Elgarøy, S. Hannestad, and T. Haugbølle, JCAP **0309**, 008 (2003); G.J. Mathews et al. astro-ph 0406046.
- [32] M. Crawford and D. Schramm, Nature **298**, 538 (1982).
- [33] Note that some authors define ε by $L = \varepsilon \dot{M}_\bullet c^2$. However, for $\varepsilon \approx 0.1$ and the inherent uncertainties of this value the difference is negligible.
- [34] This assumption is optimistic because one neglects a possible dumping of kinetic energy connected, for instance, to outflows or relativistic jets.
- [35] But note that for our purposes a subcritical growth would be sufficient, whereas the authors of [27] employ critical growth which forces them to cut off this mechanism by hand.
- [36] Note that the values cited in this section depend on the model on which the fit relies and the selection of priors and observations which are taken into account. However, all the variations which may occur due to different fits are negligible for our purposes.